Dozble-Curved Shells of Flat Quad Meshes

Summary

Each free formed shell (blob shell) can be triangulated indeed, but quad plane meshes are more economical (less cutting-waste, less joints, less buckling danger of interconnected structural panels). To avoid tall acute meshes, shells of parallelograms, called "translational surfaces" are limited to be slightly curved, in contrast to shells of trapezoids, called "scale-trans shells" that are also still restricted concerning the number and orientation of openings. My idea being applied for patents is to combine several slightly curved translational or scale-trans surfaces into one shell of nearly each demanded shape. Its net can be generated by vaulting a basic polygon or, circumscribing a basic polyhedron. These basic shapes define the curve's planes.

Keywords: *double-curved surfaces, grid-shells, panellised shells, sail-vaults, cushion-roofs, domes, blobs*

1. Introduction

The problems of a blob shell consisting of only one or two scale-trans surfaces [1].are shown in Fig. 1: There, the trapezoidal shape of the meshes is made plain by a vertical projection onto a separate horizontal plane. The right rear quadrant is a pure scale-trans region: Curves consisting of chords, having equal shapes but varying sizes, had been moved downwards. Hereby, tall triangles meeting in a polar point on the shell's rear side have been avoided, but many quads are arbitrary cut off and hereby reduced into unfavorable triangles or pentagons.

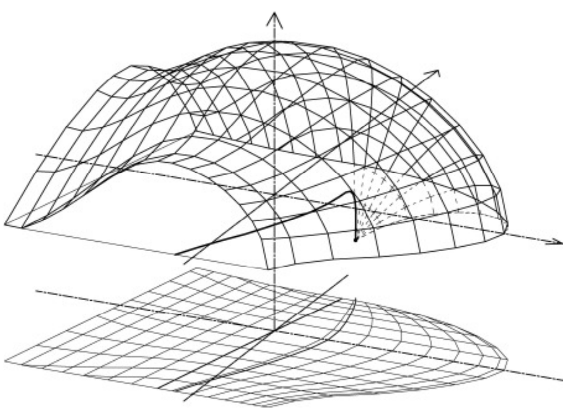

Figure 1. *A conventionally meshed blob*

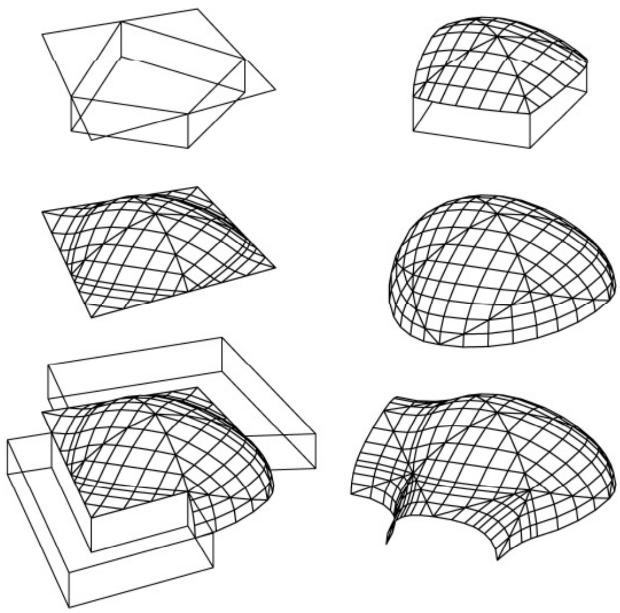

Figure 2. *Shells being meshed in a novel way*

Besides, the shaping of the border regions of both openings is interdependent: The anticlastic curvature of the left opening's region implies an inversion of the frontal free border arc's curvature, causing weakness against buckling. Even an unequal scaling of single chords cannot solve this problem.

Because of this, a solution to generate different regions or openings of a faceted shell more independently from each other had to be found. This has the positive side effect that regions of a shell can be exchanged in order to reuse them in another context, as can be seen in Fig. 2. Parts of different sorts of shell shapes like sail-vault, cushion-roof, Isler-shell, or blob can be combined in a single shell.

The blob shell in Fig. 2 (bottom, right) is an adequate solution for the problems shown in Fig. 1. This solution cannot be described before the end of a series of examples leading from flat and symmetric, to spatial and asymmetric configurations by increasing complexity. On this way, as a good spin-off effect, better solutions for flat cupolas or spherical domes are found.

The aim of this paper is to combine the versatility of a net of triangular shell's pieces having a large scale with the regularity of a net of flat quad meshes having a small scale.

2. Vaulting a Polygon

2.1 Sail-Vaults

The number of shell's pieces is small at first: In Fig. 3 (top, left) a quarter of an usual translational shell vaulting a rectangular basic polygon is copied and bisected diagonally into quad-meshed triangular shell's pieces, called "sherds". These sherds are copied and then mirrored in the plane of their red cutting line. By this, triangular meshes along the cutting line, called "cut-triangles", are fused in pairs into new quad meshes, called "fusion-meshes".

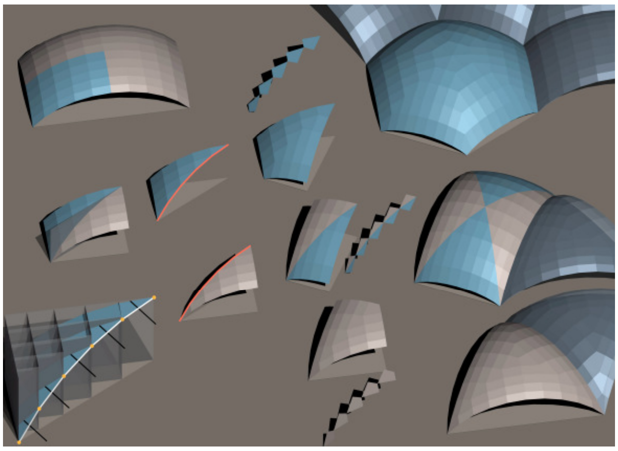

Figure 3. *composing sail vaults*

Three times in Fig. 3, two sherds form a spatial quadrangle. The quadrangle being shown in the middle is a conventional one again, whereas the upper and the lower symmetric one form a new shell's part being called "double-sherd". Such double-sherds can be combined to be part of a sail-vault covering a basic polygon, that is, a regular hexagon (top, right) and a regular triangle (bottom, right). Sail-vaults of equal or varying forms can be combined conventionally to become a vault system.

To be fused, adjacent meshes have to be coplanar The magnified sherd in Fig. 3 (bottom, left) shows black lines of altitude on each cut-triangle. Each missing adjacent triangle of another sherd must have an equally oriented line of altitude, in this simple case, normally to the mirror plane. Each of the cut-triangle's borders, being called "cutting chord" is a segment of a cutting line to be fused into a "cut-fusion-line". Here, the cut-fusion-line defines the shape of all other curves. It is the "determining curve" of each sherd and each shell of Fig. 3. Such a curve is highlighted in other pictures too. By its orange nodes, the sherd's planes of curves intersecting at right angles are defined. The planes of one curves' set have to be parallel to the border's plane that includes an edge of the basic polygon. The determining curve is a circle-arc being segmented into equal chords. The determining curves of all other examples of shells are circle arcs too, if no other characteristics are mentioned.

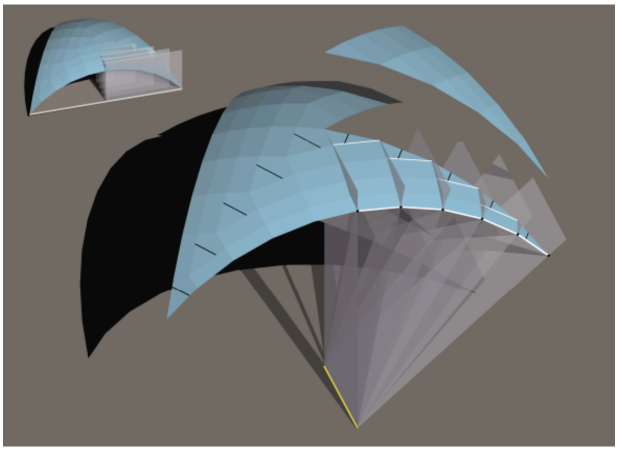

Figure 4. *A sail-vault of scle-trans surfaces*

The sail-vaults described before consist of sherds each having a translational subdivision by chords of plane curves. Figure 4 shows now a triangular sail-vault of sherds each having a scale-trans subdivision. For each sherd, there are two sets of planes again. There is still one set of parallel planes, but the planes of the other set are now disparallel: They intersect all in one horizontal central yellow line. The latter planes are visualised by opaque rectangular plates radiating from that line and penetrating the lower right-hand sherd. The advantage of this centric scale-trans subdivision is the small number of mesh patterns for mesh panels. Like on a globe of meridians and parallels of latitude, each row of one direction in such a sherd consists of equal meshes. Now, the determining curve is a shell's border being situated within a sherd's border plane being part of the set of still parallel planes being parallel to the polygon's beige edge.

2.2 Cushion-Roofs

From each synclastic sherd, a beige anticlastic sherd can be derived. In parallel, violet chords of the synclastic sherd are transferred to the resulted sherd and then cut by, or enlarged until the ground plane. By this, a horizontal straight borderline is growing as the new sherd's cutting line. Consequently, the plane of each mesh like the red one of an anticlastic sherd is parallel to that of a corresponding mesh of the neighbored synclastic translational sherd. The sequence of corresponding meshes or, chords is mirror-symmetric to the green border between both inversely curved sherds.

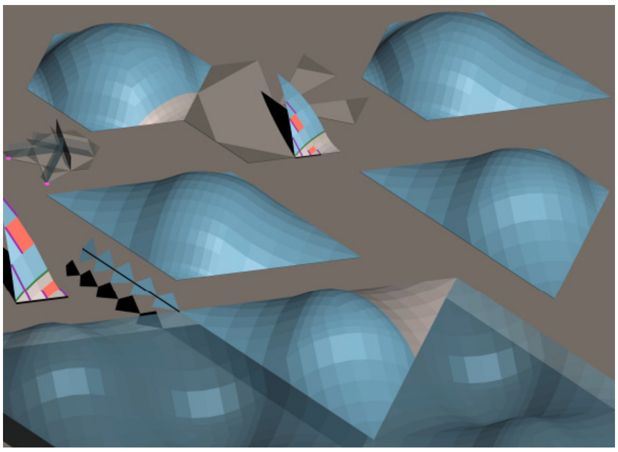

Figure 5. *Composing and varying cushion-roofs*

Each beige pair of symmetric anticlastic sherds closes an arched lateral opening of a sail-vault. If a sail-vault of Fig. 3 is enlarged on each side by such a sherds' pair, a cushion-roof of mixed curvature is generated. The surface of a beige anticlastic sherds' pair in a corner region of a cushion roof is tangential to the ground plane in the roof's corner point. The outline of the cushion-roof in plan is a flat polygon having sides being straight-lined like that of the initial sail-vault, being circumscribed by it.

Other cushion-roofs can be added to result in a continuous undulated roof like in Fig. 5 below. Along the straight borderlines, the triangular cut meshes fuse in pairs into fusion-meshes again, but the direction of stepwise rotation of the fusion-mesh's plane from one place to the next differs: The rotational axes are now aligned in the (black) cut-fusion-line itself, in contrast to these of a synclastic double-sherd that all cross its cut-fusion-line. In other words: The chain of fusion-meshes can be considered to be twisted instead of bent.

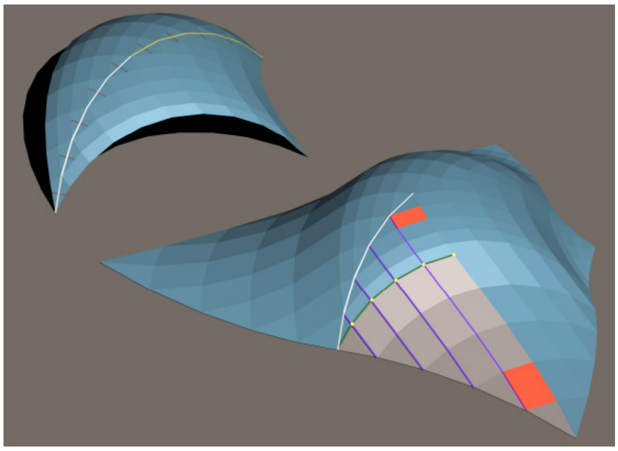

Figure 6. *A roof covers a quadrangle having curved sides.*

The border of the cushion roof in Fig. 6 is still plane, but the sides of the outline polygon are swinging, because the blue curves of the roof's translational nets are swinging. This is why both determining curves of the included sail-vault are spatial instead of, vertically plane like that one in Fig. 3: Each fusion-mesh of Fig. 3, including its cutting-chord as part of the determining curve and each black line of altitude, is slightly rotated in plan against the next fusion mesh by a constant angle. Since chords and meshes are still corresponding in pairs by parallelism, the yellow nodes of the green border-arc between both inversely curved sherds are turning-points of the blue swinging curves crossing there.

Shells having curved borders in plan are exceptions. They hardly can be combined modularly That is why determining curves have to be plane normally.

Hitherto, a straight-outlined cushion-roof was restricted to cover only following polygons: a rectangle or a lozenge, when it had been conceived as a single translational surface; a trapezoid, when it had been conceived as a single scale-trans surface. However, existing courtyards or public urban sites often have irregular borders.

The shell of Fig. 7 covers an irregular straight-lined quadrangle. Into this, the basic polygon of an included blue sail-vault had been to be inscribed. The green point exactly below the zenith is neither the intersection point of diagonals nor the gravity point of the basic polygon. Yet, in this quadrilateral case, the diagonals of its circumscribing polygon intersect there. Each plane of a sherd's border is vertical. Two times, a plane being defined by the zenith and a roof's corner point, is crossing the pink, vanishing-point-like intersection point of enlargements of two basic-polygon's edges, as you can see in Fig. 7 (top, right). The black lines of altitude are parallel to the nearest straight shell's borderline. The orange nodes of all cut-fusion-lines have equal altitudes, as if they would define the ridges of a cloister vault (bottom, left).

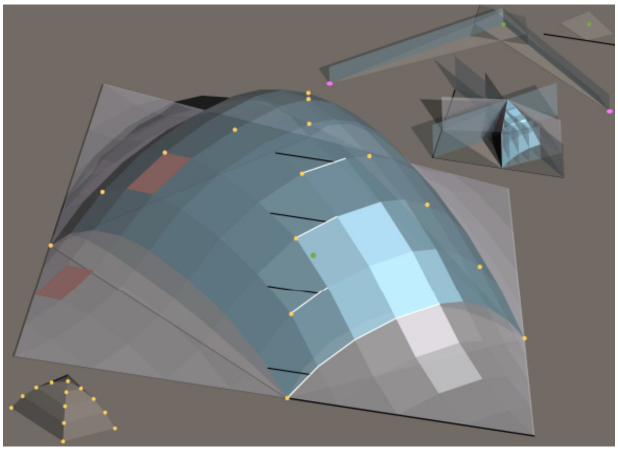

Figure 7. *A cushion-roof suited for an asymmetric context*

3. Vaulting Cubes

The shells having shapes described before can now be made completely of flat quad meshes. However, because they are still flat, they are rather roofs than buildings themselves like domes as steeply sloped symmetrical shells are. To enlarge the novel generating principle towards more spatiality, domes shall be composed of sherds too.

Two times, the upper part of Fig, 8 shows the quadratic top polygon and the upper left transparent eighth of an entire cube below. On this geometric base, a sherd of the future dome will be generated.. The sherd will vault an isoceles right triangle being an eighth of the solid cube's top quadrat.

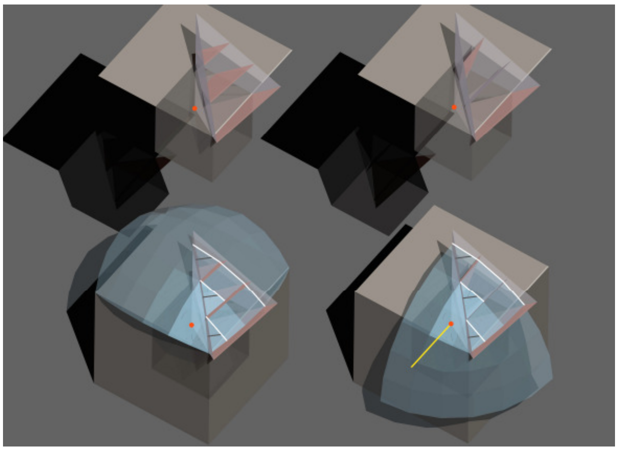

Figure 8. *Cubes define border's planes of sherds or caps.*

All three planes of the shell's borders intersect in one red reference point in the lower rear vertex of the small transparent cube being the centrepoint of the large solid basic cube. The planes of shell's borders are represented by opaque triangles. Two of them being vertical are not colored; while the third one being inclined is red colored like two other triangles representing inclined planes for the same set of curves. On the left half of Fig. 8, these inclined red planes are parallel; on the right, they intersect in a horizontal yellow line being the lower left rear edge of the small transparent cube. The plane of the white determining curve being centred in the red endpoint of this line is now perpendicular to the nearest basic-polygon's beige edge instead of including it as it does in Fig. 4.

The three shell's border's planes act like kaleidoscopic mirrors, when the first sherd is multiplied in order to result into a cap. A quadrilateral cap covers the top square of the left solid cube. In contrast to the blue quadrilateral sail-vault included in Fig. 7, the cap's border-arc's planes are not plumb to the basic polygon. Another cap on Fig. 8, being triangular, is centred in the cube's spacediagonal. In contrast to the triangular sail-vaults of Figs. 3 and 4, its border-arc's planes are not perpendicular to a (not rendered) basic equilateral triangle being a face of an octahedron here. Both new caps are not modules for vault-systems, but parts of one dome shell.

The sherd's borders of such a dome in Fig. 9 are subdivided into four chords instead of three (Fig. 8) or five (Figs. 1 – 6). The parallelism of the curve's planes facilitates the exact insertion of right-angled cut-out openings as well as partition walls and floor plates stiffening these openings. Most of the nodes of the dome's surface are not exactly positioned on a sphere. Only the determining curve of each sherd can be exactly circular, but the extremely slight deviations elsewhere can be ignored.

The novel subdivision is an useful alternative to a geodesic one. Not only cutting waste is reduced, and acute-angled triangles or lozenges are avoided. Additionally, the number of element patterns is markedly smaller. Because of the option to cut out openings being rectangular, a quad-meshed dome is very advantageous compared to a geodesic one especially when it is based on a cube [3].

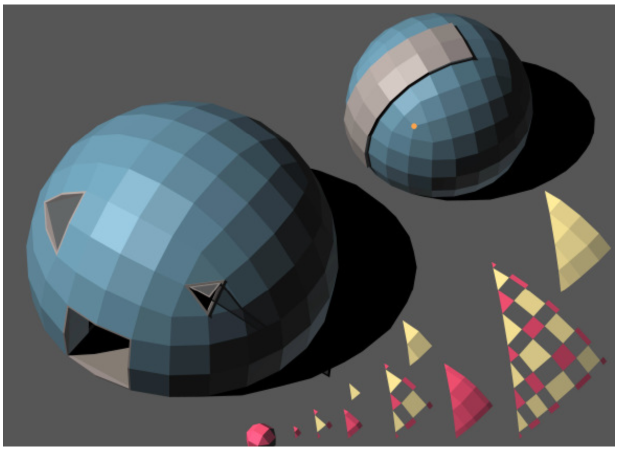

Figure 9. *A small house and a dome of an observatory*

Such a dome's sherd resembles to the yellow "fundamental region" of a red Transpolyhedron's Dual [2]. However, no one facet, that is, quad mesh of this region has parallel edges. Besides, its borders can be composed only of 2, 4, 8, 16... chords because the generation process is quiet different: It is iterative by so called explosions and an implosion.

In Fig 10 (top, left) an anticlastic sherd is derived from a synclastic one like in Fig. 5, but both sherds are based on cubes now. The anticlastic one can be considered to base on an infinite polyhedron of hollow cubes and hereby as a part of an infinite surface similar to a Schwartz-surface, dividing space into two interwoven halves like the infinite cubic polyhedron does. To form this infinite polyhedron being represented in Fig. 10 by whole, halved or quartered quadrats, two opposite faces of each hollow cube are removed. Each hereby opened side adjoins to the void space of a cube having been completely removed. This void can be refilled by a solid cube as base for additional synclastic sherds.

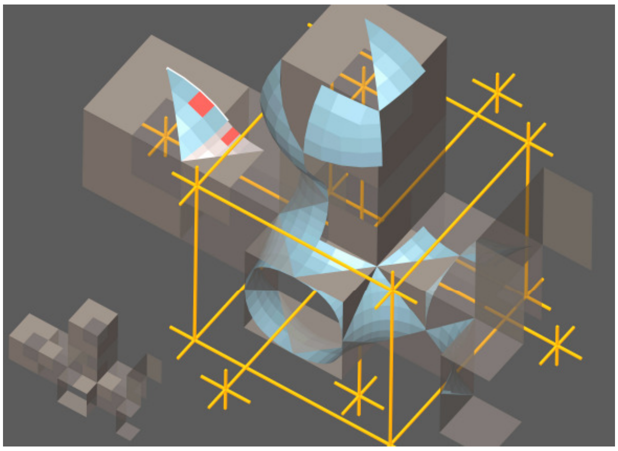

Figure 10. *Cubes as parts of an infinite polyhedron*

Fig. 11 shows a compound of domes being enlarged and connected by extensions enabling continuous transitions. Each extension, made of four anticlastic sherds of one pattern and of four of its mirrored counterpart, has a large and perpendicularly plane round opening. Even if the opening is not closed by an adjoining enlarged dome, the extension doesn't need separate stiffening elements, because it is stiff itself by its anticlastic curvature. Vice versa, Fig. 11 can be considered also as a part of an infinite anticlastic shell partly being closed by synclastic regions: Extensions are replaced either by a cap or, closed by a small halved dome whose sherds are derived also by parallelism from neighboured anticlastic sherds.

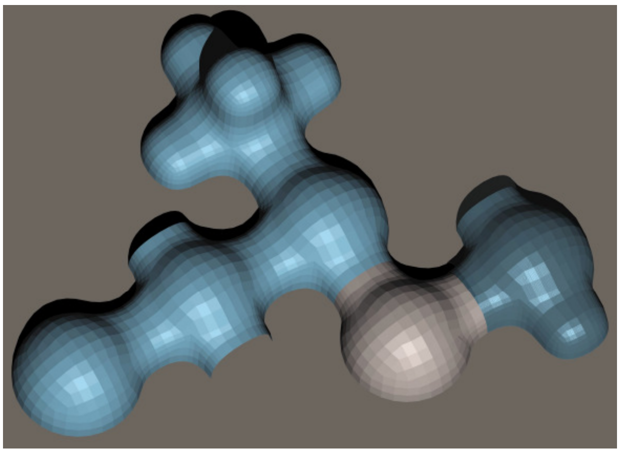

Figure 11. *a complex shell according to Fig. 10*

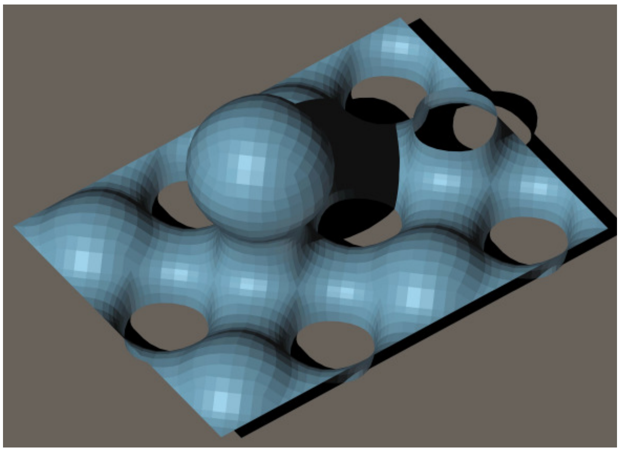

Figure 12. *A cantilevering shell according to Fig. 10*

A more spectacular building is shown in Fig. 12: A dome exceeding the extent of a hemisphere emerges from a complex shell forming courtyards and skylights made of tunnel's parts. In contrast to Fig. 11, this structure cannot be divided into single shells, because some of them would tilt in this case.

4. Transformations

A symmetrical shell can be transformed into a blob shell by: exchanging chords, shaping a determining curve deviating from a circle arc, shaping parts anew, or by scaling.

In Fig. 13 (top, left), the right darker shell of Fig. 11 is the object of a local transformation concerning seven sherds made transparent here. The resulted shell's part being more synclastic is lifted up. It has been generated by assorting all equally oriented pink chords of one blue curve running from the left to the right through several sherds, in order to result in a longer chord of a new curve having at least less turning points. The same applied in the middle of Fig. 13, showing this shell's rear side. Additionally, a transparent quadrilateral region of one open extension, consisting of four sherds each having the same two sets of parallel planes of curves, has been transformed by exchanging, within one of these sets whose planes are vertical, each pink chord of both sherds more in front by an equally oriented chord of both rear sherds. Hereby, the inclination of the other, that is, the transverse set of planes is changed from 45° to –45° to the plumb. All these transformations resulted into the blob shell in Fig. 13 (bottom, left) resembling already somewhat to the final one in Fig. 2 (bottom, right). However, the meshes are still too acute-angled; the curvature seems to have kinks exceeding the regular ones.

Nevertheless, his shell can be reused by removing the blue semi-transparent part in the front and by turning the beige part 90° clockwise. The gap of the resulted shell in Fig. 13 (top, right) is filled by a blue semi-transparent unchanged and a mirrored copy of the beige part. Alternatively, as you can see below, the gap is filled by a new large and less symmetric triangular cap being defined by the gap's green borders in vertical planes. These green curves consist neither of basket-arcs nor elliptical arcs.

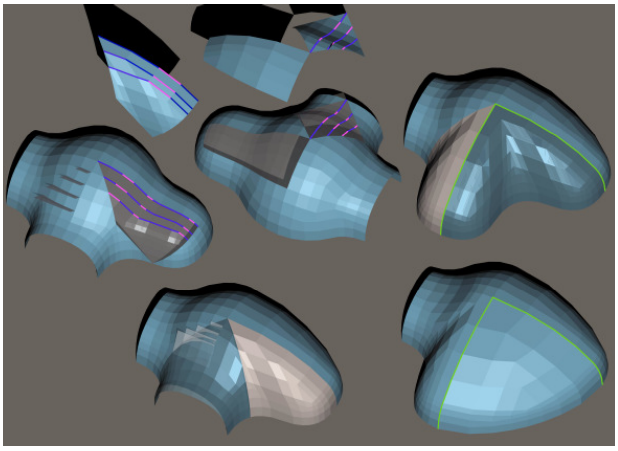

Figure 13. *Changing regions of a single shell*

The borders of a triangular transparent cap in Fig. 14 have a curvature changing its strength too, but they are more geometric, because the determining curve is a basket-arc. It is composed of a white and a yellow circle arc. Their different radii enable advantageous centric scale-trans subdivisions.

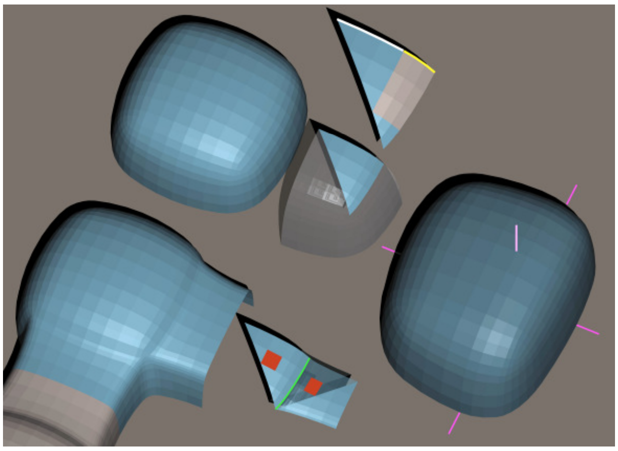

Figure 14. *Rounded cubic buildings*

The completed synclastic shell looks like a rounded box. In Fig. 14 (bottom left), it is enlarged by two open extensions, whose sherds differ recently.

The asymmetric shell in Fig. 14 (bottom right) is derived from the symmetric shell: Each triangular cap of it has been differently scaled, but it has the same x or y scale factor like an adjoining one.

The extensions of Fig. 14 are reduced in Fig. 15, but not by scaling. Instead of this, a yellow new vertical plane border for the anticlastic blue sherd rendered magnified in Fig. 15 (top. right) has been fixed as a circle arc of equal chords. Only its first chord and its pink last one are still parallel to the last and the pink first one of the white determining curve of the beige magnified synclastic sherd. Hereby, strong kinks between inversely curved sherds and between extensions, are avoided, that is, smooth transitions are maintained; but a (green) spatial cut-fusion-line had to be taken into account.

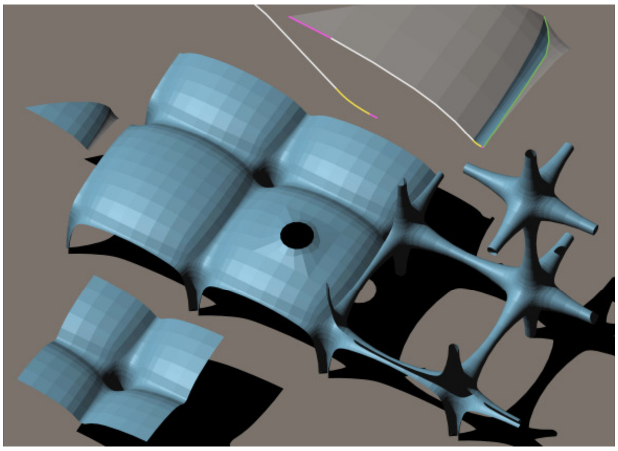

Figure 15. *Isler-like shell units and a hollow skeleton*

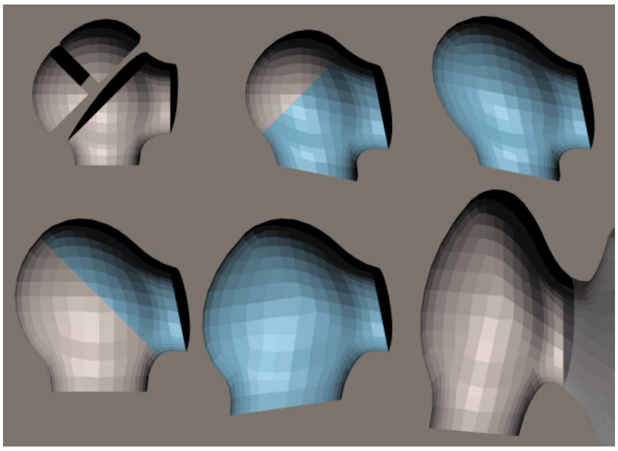

Figure 16. *Scaling of a shell in parts and as a whole*

The quadratic or cubic Isler-like units can be added as pavilions and/or cantilevering roofs. Sherds of the reduced extensions can be added to become parts of a hollow skeleton.

Fig. 16 shows a sequence of a scaling being similar to that of the shell in Fig. 14 (top, left). At first, he object of scaling, the beige shell of Fig. 11 is cut into parts now by the cut-fusion-line's planes instead of the planes belonging to the parallel's sets. The shell part's scale factors are more uniform than in Fig. 14 (bottom, right). The result of this scaling is the blue shell in Fig. 16 (bottom, middle). Additionally, this shell has been scaled finally as a whole in directions being parallel or perpendicular to the right-hand opening's plane. The resulted grey oblong blob shape on the left of it is a better approximation to that of Fig. 2 (bottom, right) than that in Fig. 13 (bottom, left).

5. Vaulting symmetric Polyhedra

5.1. Packings of Polyhedra

Infinite polyhedra dividing space into equal halves can be made of other polyhedra than of cubes like in Fig. 10, if these polyhedra are able to be densely packed likewise.

Figure 17 is based on tetrahedra and truncated tetrahedra. The beige anticlastic sherd's pattern of the blue infinite anticlastic surface's part is derived from the blue synclastic pattern by parallelism again. The infinite surface enclosing two interwoven tunnel-systems each having orange centrelines being parts of a diamond's crystal grid has less application options because there are less options to cut out useful parts by planes being perpendicular to each other. That is why the results are more unusual. The shell in Fig. 17 (bottom right) can be considered either as a barrel-vault having an enormous bump above exceeding even the ground area or, as a dome having two flat open extensions.

The parts of Fig. 17 have been rotated differently in order to use them within the shells of Fig. 18: The triangular shell (top, left) is made of three semi-transparent cushion-roofs around a non-transparent tunnel region whose centreline is vertical. This region has yellow straight-lined borders being the outlines of the top and bottom polygon of an imagined octahedron being oriented as an antiprism.

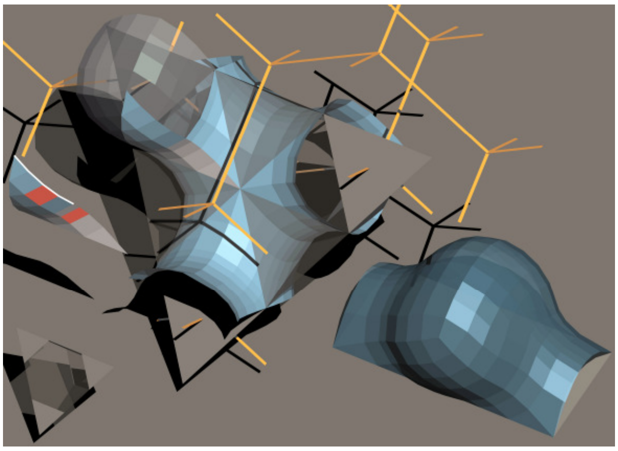

Figure 17. Shells based an tetrahedra and truncated tetrahedra as parts of an infinite polyhedron

The less symmetrical shell (bottom. left) has two trumpet-like, but angular shaped, enlarged extensions having openings in planes being inclined to the ground. Its third opening on the left has been changed by replacing the uniform blue anticlastic sherds by beige ones of a new pattern and its mirrored counterpart, enabling not only an useful extension whose opening's plane is vertical again here, but also plane shell's borders on the ground.

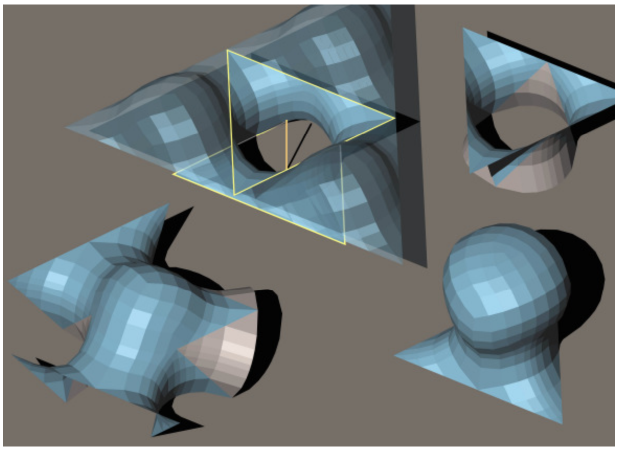

Figure 18. *Sculptural shells using parts of Fig. 17*

Fig. 19. is based on octahedra that, together with cuboctahedra, could be densely packed. The grids of tunnel-centrelines are cubic again, but they are rotated 45° around a horizontal axis running from the left to the right. The grids' nodes don't coincide with the potential centrepoints of synclastic shell regions like the rendered triangular semi-transparent cap. The anticlastic sherds belong to an infinite surface that is topologically equivalent to that of Fig. 10, but they have other proportions.

The results shown in Figs. 20 and 21 are unusual too. Here, the initial orientation of the shell's parts has not been changed. Hereby, the triangular cushion-roof in Fig. 20 is in a sloped position to the horizontal ground. The semi-transparent shell neighbouring on the left resembles to that in Fig. 17 (bottom, right), but its bump above is flatter. Because their open extensions are flat, all shells of Figs. 20 and 21 are restricted to be only roofs.

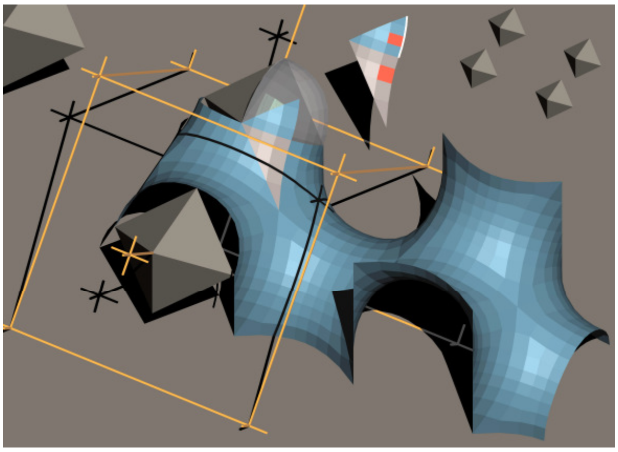

Figure 19. *Octahedra as parts of an infinite polyhedron*

The beige part of a shell in Fig. 20 is reused in triplicate in Fig. 21 (top, left) in order to form a beige roof having northward skylights. Besides in Fig. 21 (top, right), the blue semi-transparent shell of Fig. 20 had been scaled and tripled in order to result in a blue undulated flat barrel-vault. The initial form hardly can be recognised, because the scale factors of both scaling directions were so enormously different that the former oblong side is now the transverse side of the single shell as a module.

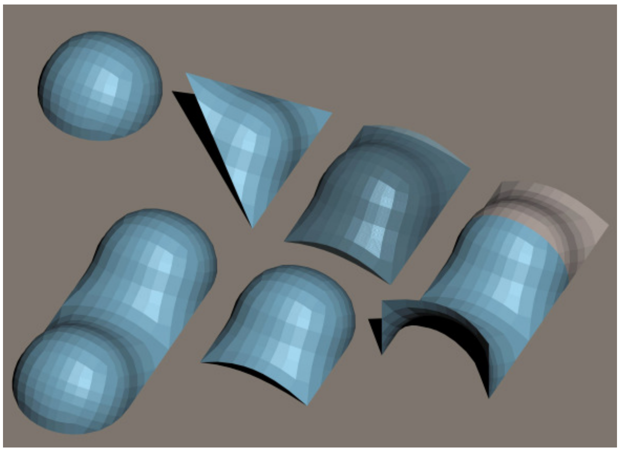

Figure 20. *Shells using parts of Fig. 19*

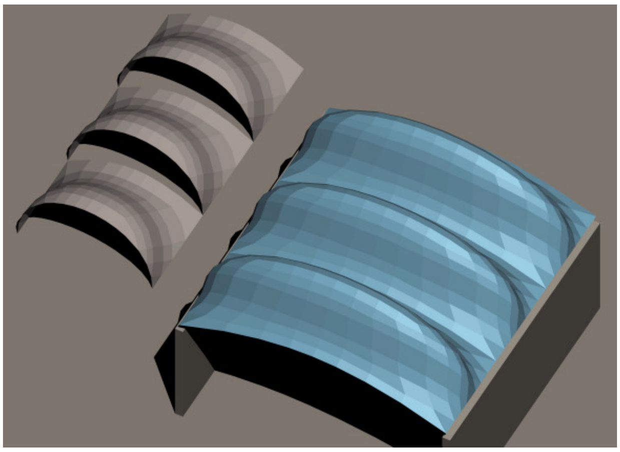

Figure 21. *Shell's parts of Fig. 20 being multiplied and/or scaled to be a part of an undulated barrel vault*

5.2. Larger Symmetric Polyhedra

Despite a cube is the most applicable basic polyhedron, other very symmetric polyhedra like Platonic, Archimedian, or geodesic polyhedra can be the geometrical base for novel spherical solitary shells being suited to replace geodesic domes.

An example of a Platonic polyhedron is the dodecahedron in Fig. 22 (bottom, left). Only one magnified transparent eighth of it (top, left) having been cut out along three planes being perpendicular to each other serves as a base for one synclastic sherd's pattern and two differing anticlastic ones.

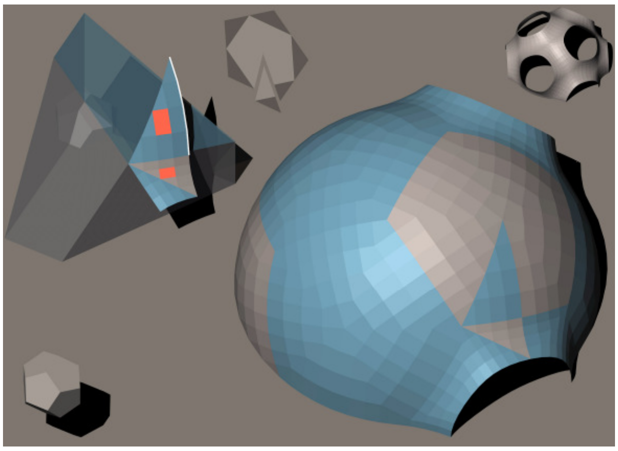

Figure 22. *A dome based on a pentagonal dodecahedron*

Here again, the border's planes of the synclastic sherds and the anticlastic double-sherds are kaleidoscopically mirroring these sherds in order to form a dome shell. Optionally, all other pentagonal entire or halved caps could be replaced by extensions in order to form additional skylights or openings.

The soccer-ball-like truncated icosahedron in Fig. 23 is an Archimedian Polyhedron. The sherd of the hexagonal cap has been generated at first, because it is larger than that of the pentagonal one. The deviations of the nodes from the spherical surface are smaller in a smaller cap.

The green half border-arc of the hexagonal cap is only a resulted one; but for its part, it determines then the semi-transparent pentagonal cap. Only the cap generated at first can have a centric scale-trans subdivision saving mesh patterns, because no one curve of the second cap is equidistantly subdivided. The fusion-meshes' black "lines of altitude" are still parallel to the plane of their respective basic polygon, but these planes are sloped or even vertical.

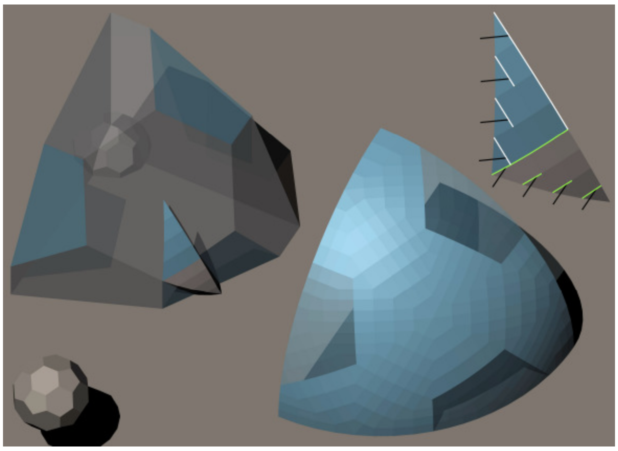

Figure 23. *A dome part based on a truncated icosahedron*

The stellated icosidodecahedron in Fig. 24 (bottom, left) is a geodesic polyhedron acting itself as a basic polyhedron. It is composed of equilateral triangles and flat pentagonal pyramids, whereas each pyramid consists of five isoceles triangles.

A cap vaulting a basic isoceles triangle needs three different sherd's patterns and three mirrored counterparts. Its zenith is situated on a straight line running from the shell's centrepoint through the green gravity point of an isoceles triangle. In contrast to all other caps before, this line is not perpendicular to the penetrated basic polyhedron. Two of such lines define the plane of the sherds' green border, including the red endpoint of a yellow second determining curve. The first, white curve determines the sherd of the equilateral cap in same time.

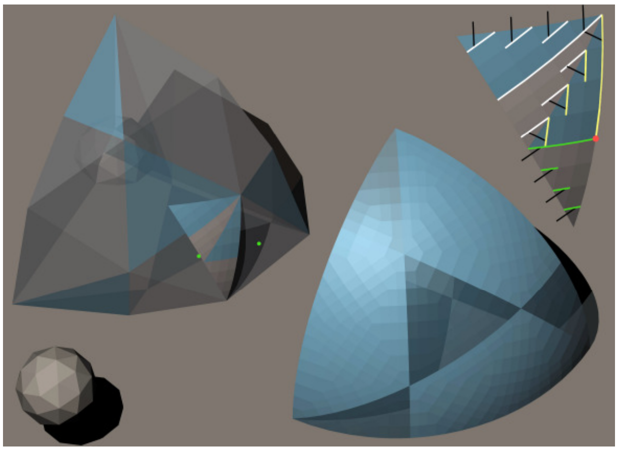

Figure 24. *A dome part based on a geodesic polyhedron*

5.3 Prisms

Like a cube, prisms have vertical lateral faces. The sherd's formats of the top cap and the lateral cap have to differ in each case, because the lateral and the top polygon differ now The basic prisms are rendered halved horizontally by the ground plane.

The basic prism in Fig. 25 is formed by reducing the number of side polygons being still quadratic from four to three. The anticlastic sherds are reduced to form open extensions like the border regions of an Isler shell. Red meshes are corresponding in pairs by parallelism again. Like in Fig. 15, a yellow curve determines the upper anticlastic sherd of the Isler-like border region, but now, each pink chord of the yellow curve has been fixed to be parallel to the corresponding pink chord of the white determining curve. In contrast to all precedent examples of parallelism, its length is exactly halved.

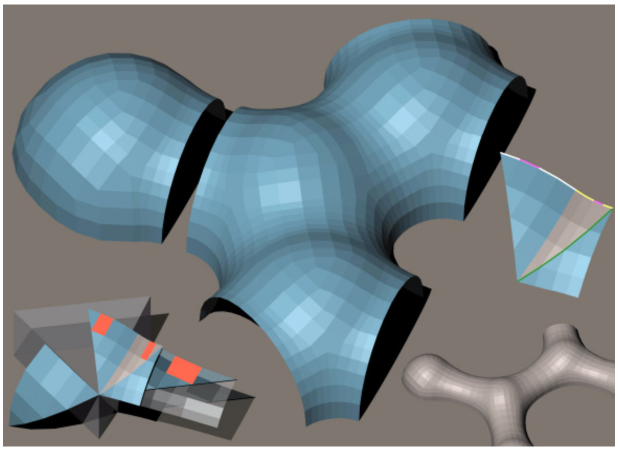

Figure 25. *Dome and Isler-like shells around prisms*

In Fig. 25 (bottom, left), below the reduced anticlastic double-sherd, there is an unreduced transparent double-sherd too, whose rear blue coloured sherd can be a part of a triangular cushion roof. Such double-sherds form the diminished beige tunnel-system in Fig. 25 (bottom, right), having a planar hexagonal grid of tunnel-centrelines.

The asymmetry of prisms can be increased. In analogy to Fig. 3, but now based on prisms, several modularly sectored shell's parts of Fig. 26 can be combined to result in a plurality of varying shells being shown in Fig. 27. These shells are not flat, but steeply sloped. Each different basic polygon is now the top polygon of a prism. Like a regular basic polygon, each prism consists still of modular sectors being rendered here as alternating between solid and opaque. Their angles are now those occurring in the well known quadrat and in a regular pentagon, instead of a triangle or hexagon in Fig. 3.

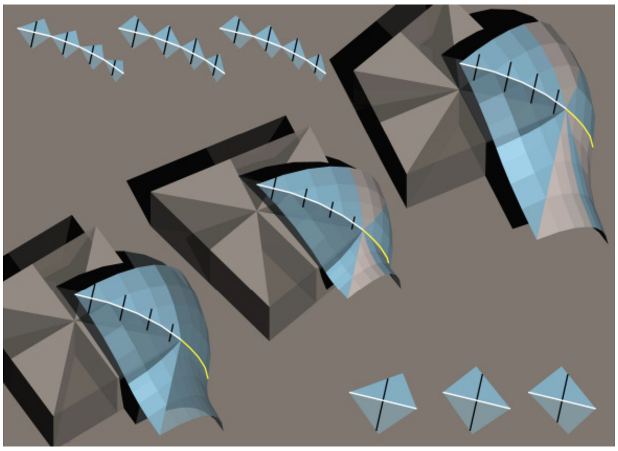

Figure 26. *Six sectored shell's part's patterns for Fig. 27*

By combining top cap's sherds, even if they base on different modular prism sectors, double-sherds and their fusion-meshes are no longer restricted to be symmetric. All determining cut-fusion-lines are congruent. The fusion-meshes' lines of altitude are still oriented normally to the planes of these curves.

Each of the cushion-roofs in the first row of arrayed shells in Fig. 27 consists only of the upper part of the shown shell's parts, being located only above the top polygon. Each shell of a column of arrayed shells has an equal top cap. A shell of the middle row differs from the shell below by the number of openings. The top polygon of the basic prism is, from the left to the right: a pentagon, a kite, a rectangle, an asymmetric, and a symmetric trapezoid.

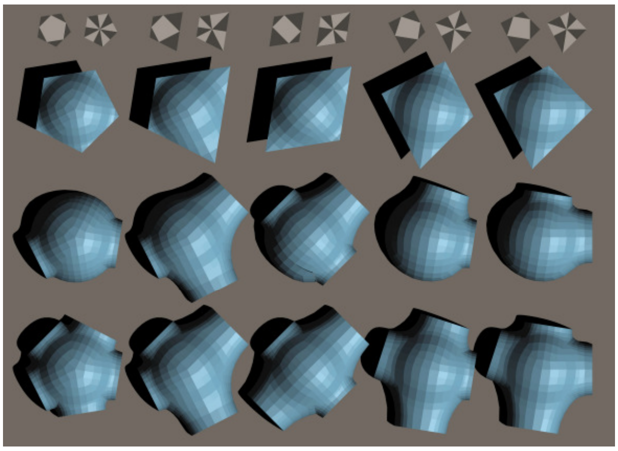

Figure 27. *Shells made of a few sorts of shell's parts*

6. Vaulting an Irregular Polyhedron

To achieve a pure blob surface, the following small step from an irregularly trapezoidal prism to an irregularly quadrilateral one as a basic polyhedron had to be done. This basic solid can be seen separately diminished in Fig. 28 (top, left).

The generating process of the top cap in Fig. 28 combines characteristics of that in Figs. 7 and 8. According to Fig. 8, the red planes of one of each sherd's both sets of crossing curves are inclined instead of being parallel to each other and, instead of being perpendicular to the ground like in Fig. 7. The red planes converge again in order to intersect all in one yellow horizontal straight line. According to Fig. 7, the planes of each of both planes' sets of a potential cushion-roof's anticlastic sherd are parallel to each other. However now, the planes of one of them, rendered here as grey triangles in the front, are inclined in parallel to a plane that includes not only the red meeting point of all shell's border's planes as reference point below the zenith, but also a beige line of the top polygon, being an edge of the basic polyhedron. Parts of the synclastic sherd's resulted green borderline have been transferred in order to end in blue points of the black straight cutting line. These points are situated within the vertical planes of the synclastic sherd's curves.

For the anticlastic sherd, the translational subdivision has been selected, because it looks more regular there. That is why small oblong tapered quad meshes are avoided that could occur either by transferring each chord in parallel from one existing scale-trans sherd to the new other one or, by fixing additional inclined radiating planes on the blue points, instead of parallel ones. Within a synclastic sherd, a scale-trans subdivision looks more regular, because the meshes are less acute-angled.

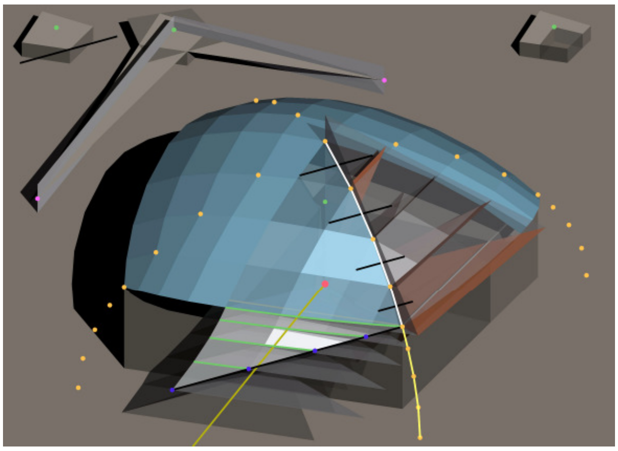

Figure 28. *Cap of the blob shell and adjoining sherds*

Both determining curves (white, yellow) are situated within a vertical plane including the zenith and the top polygon's vertex in the front on the right. These curves are derived from two coplanar sherds' borders of a the beige shell in Fig. 11 based on a cube, by scaling them horizontally only, keeping hereby the altitude of their nodes. Like in Fig. 7, the orange nodes of different curves have equal altitudes. The white, upper determining curve is the cut-fusion-line between the opaque blue and the transparent sherd. Its coplanar continuation below by a yellow, second determining curve can be seen in Fig. 29 as the left border of the beige right-hand lateral half cap.

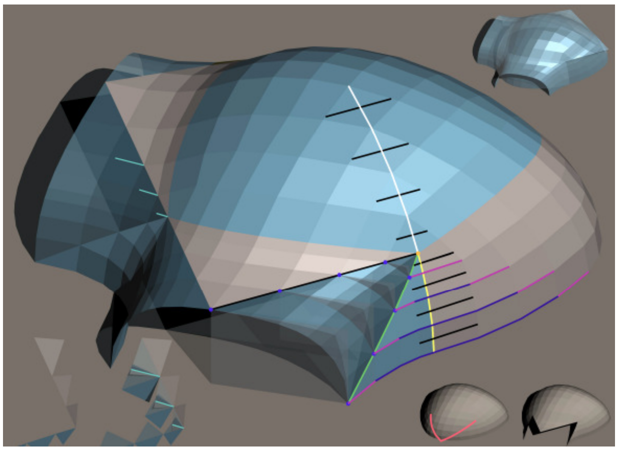

Figure 29. *The final blob shell having open extensions*

Fig. 29 shows the complete final blob shell being identical to that of Fig. 2 (bottom, left). In Fig. 29 (top, left) you can see a superimposed arrangement of all shell's parts forming the shell variations of Fig. 2.

To derive a frontal extension's lower lateral anticlastic sherd from a synclastic one in Fig. 29, pink chords of blue curves within common horizontal planes correspond in pairs by parallelism. Each blue node of this anticlastic sherd's green cutting line being spatially curved has to be situated in a vertical plane being oriented in parallel to the basic polyhedron's front polygon and being fixed on another blue node of the potential cushion-roof's sherd's horizontal borderline being straight further on. This black straight line is not a resulted one, but a determining one. If it would have been turned slightly clockwise around the upper right frontal vertex of the top polygon, the frontal extension would have become less deep, while the left lateral extension would have become deeper. The shape of the whole shell would have been stretched within the direction from the left to the right.

At last, the semi-transparent blue sherds filling wedge-shaped gaps have been generated. These gaps can be imagined as a result of plastically deforming (bottom right) a beige shell's weak frontal cap having been cut twice along a red cutting line before, whose resulted three parts are then dragged apart from each other in order to become anticlastic.

However in detail, these blue semi-transparent sherds are generated as shown on the shell's left extension: The turquoise lines had to be found, as can be seen in Fig. 29 (bottom, left): Three lower and three upper copied cut-triangles have been doubled to result into parallelograms. Then, each lower parallelogram was copied and moved to the respective upper one and attached on a turquoise common node there. Each intersecting line of two parallelograms is a turquoise line being part of a curve of one set of curves of the blue semi-transparent sherds.

7. Materialisation

Each previous example of a faceted shell is shown only as a thin surface of flat areal meshes without any thickness, similar to metal sheets. However, thickness is necessary indeed, when each mesh, as a structural insulated panel, shall transfer completely the loads of the shell as a self-supporting envelope.

A panellised blob shell's structural thickness is shown in Fig. 30 (top, left) by the red-coloured free border-arc in the front. The same view below shows a grid-shell according to the same interdependent outward and inward surfaces.

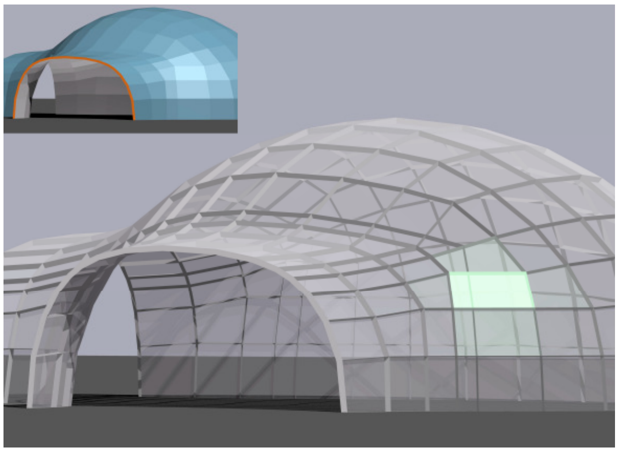

Figure 30. *A panellised structure and a grid shell*

In Fig. 31, thick virtually transparent panels are mitred in order to achieve exact node situations without offsets. Each node of the inward shell surface is connected to a respective node of the outward shell surface by a short straight line. The corners of either four thick panels or of four linear elements like flat bars represented by the oblong grey edge-faces can meet there. These edge-faces are generated starting on the zenith The first short line on the zenith is situated within a long vertical line above the red reference point.

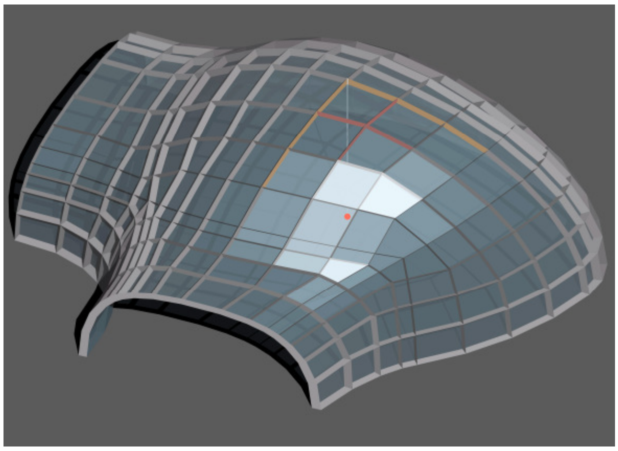

Figure 31. *A blob shell showing constructional thickness*

Within each of the vertical planes intersecting in this long line, the orange edge-faces along a curve of chords are coplanar and symmetrically mitred by the short lines - in contrast to the other edge-faces (red) out of these vertical planes, excepted these of the ground plane. A disadvantage of this arrangement of edge-faces is that each thick panel has a slightly differing thickness from another one.

In a free-formed shell, no one single panel belongs to the same pattern like any other one. Usually, this would cause confusion on site. Fortunately, each described shell can be assembled in rows being closed like rings. Hereby, one single number on each panel is sufficient to describe its final position. Fig. 33 shows these rings or rows. The assembling process could begin on the ground by a small cap of four panels around the zenith. This cap could be lifted by a crane or a column. The shell would increase and be lifted row by row without a centring or a scaffold.

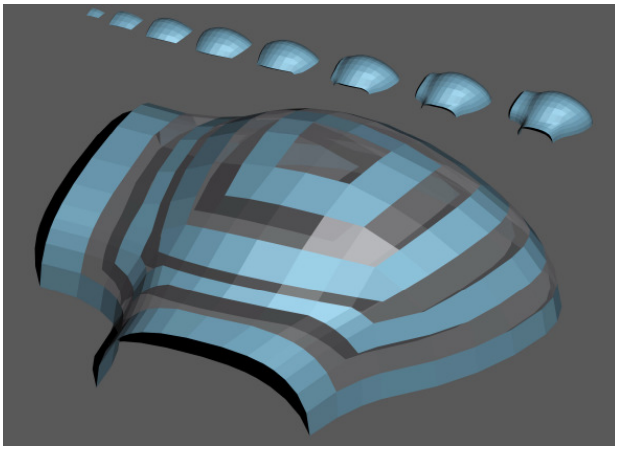

Figure 32. *The final blob surface consisting of rings*

8. Conclusions

By increasing the asymmetry and the spatial complexity of the illustrated examples on the one hand, and by diminishing the degree of symmetry on the other, finally an exclusively quad-meshed blob shell in Fig. 32 being robust by its intended shape in contrast to Fig. 1 was generated. The next step of complicating could be to incline the basic prism's lateral faces. By this, the prism would become a truncated pyramid or a parallelepiped.

It would be interesting to search, if there are still shell shapes which cannot consist of flat quad meshes even by a combination of the new subdivision with conventional ones. The challenge for an architect using the novel subdivision is to find the right basic shape and to fix several parameters fitting to the intended shell shape.

A software generating automatically a quad-meshed blob surface by initially given or interactively changeable parameters (basic and circumscribing polygon(s), basic polyhedron (polyhedra), centre-point(s), determining curve(s), degree of subdivision, translational or scale-trans subdivision) has still to be developed.

Indeed, Catmull-Clark subdivision surfaces often applying in computer software have a topologically identical mesh pattern on a basic-polygon's convex trivalent vertex situation like on the orange point in Fig. 9. However, the meshes are in general not flat and in parts very acute-angled [4].

The structural analysis of the described faceted shells seems to be not even trivial, but simple scale models have proven their rigidity as a plate structure.

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